**TASK 8: COMPETITION**

**In-class investigation**

**Unit 3**

**Topic 3.3: Graphs and networks**

**Competition**

**In-class investigation**

**Solutions and marking key**

**Question 1(a)**

|  |  |
| --- | --- |
| Solution | |
| K S  R J  A N | |
| Mathematical behaviours | Marks |
| * Identifies edges which are undirected and travel between nodes * Makes the correct number of connections | 1  1 |

**Question 1(b)**

|  |  |
| --- | --- |
| Solution | |
| Each element equal to 1 represents an edge drawn from the vertex on the left (row headings) to the vertex on the right (column headings) | |
| Mathematical behaviours | Marks |
| * describes the element equalling 1 as representing an edge * describes source and destination of edges | 1  1 |

**Question 1(c)(i)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * draws bipartite graph with labelled nodes * completes all connections between nodes | 1  1 |

**Question 1(c)(ii)**

|  |  |
| --- | --- |
| Solution | |
| Matrix could also be labelled in reverse. | |
| Mathematical behaviours | Marks |
| * labels rows and columns * determines correct entries for matrix | 1  1 |

**Question 1(d)**

|  |  |
| --- | --- |
| Solution | |
| Square matrix with dimensions = *n*  All entries = 1 | |
| Mathematical behaviours | Marks |
| * identifies dimensions of matrix * determines elements of matrix | 1  1 |

**Question 2(a)**

|  |  |
| --- | --- |
| Solution | |
| Lou, Ali and Barb | |
| Mathematical behaviours | Marks |
| * reads table | 1 |

**Question 2(b)**

|  |  |
| --- | --- |
| Solution | |
| 15 | |
| Mathematical behaviours | Marks |
| * interprets table | 1 |

**Question 2(c)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * draws bipartite graph with labelled nodes * completes all connections between nodes | 1  1 |

**Question 2(d)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * completes connections to represent matches unplayed * excludes all connections for played matches | 1  1 |

**Question 2(e)**

|  |  |
| --- | --- |
| Solution | |
| (i) (ii) | |
| Mathematical behaviours | Marks |
| * labels rows and columns of both matrices * represents connections for played matches * represents connections for unplayed matches * matrices match bipartite graphs | 1  1  1  1 |

**Question 2(f)**

|  |  |
| --- | --- |
| Solution | |
| It is a 5 x 5 matrix in which all the elements = 1 | |
| Mathematical behaviours | Marks |
| * identifies matrix dimensions * identifies all elements | 1  1 |

**Question 3(a)**

|  |  |
| --- | --- |
| Solution | |
| 4 | |
| Mathematical behaviours | Marks |
| * interprets adjacency matrix | 1 |

**Question 3(b)**

|  |  |
| --- | --- |
| Solution | |
| 19 | |
| Mathematical behaviours | Marks |
| * interprets adjacency matrix | 1 |

**Question 3(c)**

|  |  |
| --- | --- |
| Solution | |
| Add up all the elements | |
| Mathematical behaviours | Marks |
| * identifies features of adjacency matrix | 1 |

**Question 3(d)**

|  |  |
| --- | --- |
| Solution | |
| Rod, Barry and Terry | |
| Mathematical behaviours | Marks |
| * interprets adjacency matrix | 1 |

**Question 3(e)**

|  |  |
| --- | --- |
| Solution | |
| Stefan  All entries are 1 in the complementary matrix. These would be 0 in the matrix for matches played | |
| Mathematical behaviours | Marks |
| * identifies non-player * identifies all entries of 1 * identifies entries of 1 for non-played equal to 0 for matches played | 1  1  1 |

**Question 3(f)**

|  |  |
| --- | --- |
| Solution | |
| Stefan | |
| Mathematical behaviours | Marks |
| * identifies disconnected node from adjacency matrix | 1 |

**Question 4(a)**

|  |  |
| --- | --- |
| Solution | |
| The nodes fall into two groups where there are no connections between nodes within the two groups, only from one group to another. The groups are KRABDT and SJNFCP | |
| Mathematical behaviours | Marks |
| * identifies nodes exist in two groups * explains connections between and within groups | 1  1 |

**Question 4(b)**

|  |  |
| --- | --- |
| Solution | |
| C | |
| Mathematical behaviours | Marks |
| * interprets bipartite graph | 1 |

**Question 4(c)**

|  |  |
| --- | --- |
| Solution | |
| 10 | |
| Mathematical behaviours | Marks |
| * identifies features of bipartite graph | 1 |

**Question 4(d)**

|  |  |
| --- | --- |
| Solution | |
| (i) Kyle, Nigel  (ii) Rod, Barry, Dong, Fraser, Stefan, Peter | |
| Mathematical behaviours | Marks |
| * interprets bipartite graph of winners * identifies 4 losers * identifies further 2 losers | 1  1  1 |

**Question 4(e)**

|  |  |
| --- | --- |
| Solution | |
| The matrix would be 12 x 12  0 represents a loss as well as a match not played  1 in row *i* column *j* means the person in row *i* beat the person in row *j* | |
| Mathematical behaviours | Marks |
| * identifies any two features of the adjacency matrix | 1 each |

**Question 5(a)**

|  |  |
| --- | --- |
| Solution | |
| Lily and Nora | |
| Mathematical behaviours | Marks |
| * interprets data in table | 1 |

**Question 5(b)**

|  |  |
| --- | --- |
| Solution | |
| Deidre | |
| Mathematical behaviours | Marks |
| * interprets data in table | 1 |

**Question 5(c)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines correct number of nodes and connections * determines correct directions | 1  1 |

**Question 5(d)**

|  |  |
| --- | --- |
| Solution | |
| The graph is not bipartite  The nodes do not fall into two separate groups | |
| Mathematical behaviours | Marks |
| * concludes correctly * explains conclusion | 1  1 |

**Question 5(e)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * identifies 5 x 5 matrix with labelled rows and columns * enters all elements | 1  1 |

**Question 5(f)**

|  |  |
| --- | --- |
| Solution | |
| All elements are 0 because the players do not play against themselves | |
| Mathematical behaviours | Marks |
| * relates features of matrix to context | 1 |

**Question 5(g)**

|  |  |
| --- | --- |
| Solution | |
| The matrix is symmetrical about the leading diagonal in that where there is a 0 in row *i* column *j* there is a 1 in column *i* row *j* [not in leading diagonal]  If player 1 beats player 2 (1 in matrix) then the symmetrical position for player 2 beating player 1 is a zero | |
| Mathematical behaviours | Marks |
| * describes the symmetry of the adjacency matrix * explains the symmetry of the adjacency matrix | 1  1 |